

# Illusion Media: Generating Virtual Objects Using Realizable Metamaterials

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## Abstract

We propose a class of optical transformation media, illusion media, which render the enclosed object invisible and generate one or more virtual objects as desired. We apply the proposed media to design a microwave device, which transforms an actual object into two virtual objects. Such an illusion device exhibits unusual electromagnetic behavior as verified by full-wave simulations. Different from the published illusion devices which are composed of left-handed materials with simultaneously negative permittivity and permeability, the proposed illusion media have finite and positive permittivity and permeability. Hence the designed device could be realizable using artificial metamaterials.

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Since Pendry *et al.* and Leonhardt proposed an interesting idea to design invisibility cloaks [1,2], more and more attention has been paid to the cloaks and other optical transformation devices [3-16]. Recently, Lai *et al.* have presented a concept of illusion optics: making an object with arbitrary shape and material properties appear exactly like another object of some other shape and material makeup [13]. Invisibility cloak can be regarded as generating an illusion of free space [1-5]. Using the transformation optics, they designed an illusion device consisting of two distinct pieces of metamaterials, which are called *complementary medium* and *restoring medium*. However, the complementary medium was obtained from the transformation optics of folded geometry. Hence it is composed of left-handed materials with simultaneously negative permittivity and permeability. As a result, the proposed illusion device is extremely demanding of material parameters, and the applications are possible to remain in the realm of theory [18].

In this letter, we present a class of optical transformation media which can make the enclosed actual object invisible and generate one or more virtual objects at the same time. An arbitrary object enclosed by such a medium layer appears to be one or more other objects of arbitrary shapes and material makeup. As applications, we design an illusion device, which transforms an actual object into two virtual objects. Such a device exhibits unusual electromagnetic (EM) behaviors as verified by full-wave simulations. Here, we try to make the illusion media be fairly realizable. Unlike the published illusion devices which are composed of left-handed materials [13], all permittivity and permeability components of the proposed illusion media are finite and positive. Hence the presented approach makes it possible to realize the illusion media using artificial metamaterials. The principle behind such illusion media is not only bending EM waves around the actual object, but also generating one or more virtual objects within the virtual boundary.

An intuitive schematic to illustrate the proposed method is illustrated in Fig. 1. A golden apple (the actual object) is enclosed with an illusion medium layer, as shown in Fig. 1(a). Such a layer of illusion media makes any detector outside the virtual boundary (the dashed curves) perceive the scattering fields of two green apples (the virtual objects, shown in Fig. 1(b)) instead of one golden apple. In the other word, the illusion medium layer makes the EM fields outside the virtual boundary in both the physical and virtual spaces exactly the same, regardless the direction of the incident waves. The illusion medium layer has two functions, concealing the optical signature of the golden apple and generating the image of two green apples. The coordinate transformation of the illusion medium layer is similar to that of an invisibility cloak, but the illusion space contains some virtual objects with given EM parameters instead of purely free space. The permittivity and permeability tensors of the illusion media are calculated by

$$\overline{\varepsilon'} = \Lambda \overline{\varepsilon} \Lambda^T / \det(\Lambda), \quad \overline{\mu'} = \Lambda \overline{\mu} \Lambda^T / \det(\Lambda), \quad (1)$$

in which  $(\overline{\varepsilon}, \overline{\mu})$  and  $(\overline{\varepsilon'}, \overline{\mu'})$  are the constitutive tensors in virtual space (illusion space) and the physical medium layer, respectively, and  $\Lambda$  is the Jacobian transformation matrix with components  $\Lambda_{ij} = \partial x_i / \partial x_j$ , corresponding to the mapping from the illusion space to the illusion medium layer.

Similar to invisibility cloaks, the EM fields in the illusion medium layer can be found from the transformation optics [1,2] as  $\mathbf{E}' = (\Lambda^T)^{-1}\mathbf{E}$  and  $\mathbf{H}' = (\Lambda^T)^{-1}\mathbf{H}$ , where  $\mathbf{E}$  and  $\mathbf{H}$  are electric and magnetic fields in the virtual space, respectively. Because the boundary  $s_2$  is mapped to itself during the transformation  $\Lambda$ , we have  $\mathbf{E}'_t = \mathbf{E}_t$  and  $\mathbf{H}'_t = \mathbf{H}_t$ , where the subscript  $t$  indicates transverse components along the surface  $s_2$ . In another word, the tangential components of the EM fields on the whole virtual boundary ( $s_2$ ) are exactly the same in the physical and virtual spaces. Hence, by the uniqueness theorem, the EM fields outside are also exactly the same. Any detector outside the illusion medium layer will perceive EM waves as if they were scattered from the illusion object (two green apples and nothing else), and thus an illusion is generated. In the design of invisibility cloaks, the illusion space is only free space, hence there were almost no scattered fields from a perfect cloak. For the illusion media, however, the illusion space may contain many illusion objects of our choice, thus the scattered fields are the same as those from the virtual objects.

In order to demonstrate the design of illusion medium layer, we take an illusion device as example and make full-wave simulations based on the finite element method. We consider an illusion medium layer which transforms a metallic square cylinder to two dielectric square cylinders. In the following simulations, we consider the case when a transverse-electric (TE) polarized plane wave is incident upon an illusion medium layer, hence there exists only  $z$  component of electric field. The case is very similar when a transverse-magnetic (TM) wave is incident.

Figure 2 illustrates the numerical results of scattered electric fields for an illusion medium layer which transforms one metallic square cylinder to two smaller dielectric square cylinders. In order to illustrate the scattering character of objects better, we have plotted the scattered fields instead of total fields here and next. The plane waves are incident horizontally from the left to the right at 12 GHz. Figs. 2(a) and 2(c) show the scattered patterns of the single metallic square cylinder without the illusion medium layer and two dielectric square cylinders with  $\varepsilon_r = 3$  and  $\mu_r = 2$ , respectively. When enclosed by the illusion medium layer, the scattered pattern from the single metallic square cylinder will be changed as if there were two smaller dielectric square cylinders. This can be clearly observed by comparing the scattering-field pattern of the metallic cylinder coated by the illusion medium layer shown in Fig. 2(b) with that of two dielectric cylinders shown in Fig. 2(c). The field patterns are exactly the same outside the virtual boundary. Inside the virtual boundary, the field patterns in Figs. 2(b) and 2(c) are different.

Unlike the earlier-proposed illusion devices which consist of two distinct pieces of metamaterials - the complementary medium with double negative constitutive parameters and the restoring medium [13], the illusion medium layer in Fig. 2(b) is only composed of one layer of inhomogeneous and anisotropic medium. To design such an illusion medium layer, we first consider a three-dimensional (3D) one obtained by rotating the mirror-symmetric square around the  $x$ -axis. We construct the transformation for each side in the

Cartesian coordinate system, which can be expressed as

$$\frac{x'}{x} = \frac{y'}{y} = \frac{z'}{z} = k(x, y), \quad (2)$$

where  $k(x, y) = k_0 - c/(ax + by)$  and  $k_0$  is a compression factor, i.e., the ratio of the sizes of actual and virtual spaces. We have assumed that the corresponding side of polygonal metallic cylinder be expressed as  $ax + by + c = 0$ . In this example, the side length of each virtual object is 0.02 m and  $k_0 = 0.5$ . Based on the above transformation, the permittivity tensor of the illusion medium layer can be calculated. We can obtain two-dimensional (2D) parameters by setting  $z = 0$ ,

$$\varepsilon'_{xx} = \varepsilon_r(\lambda_{11}^2 + \lambda_{12}^2)/\lambda_{33}, \quad (3)$$

$$\varepsilon'_{yy} = \varepsilon_r(\lambda_{21}^2 + \lambda_{22}^2)/\lambda_{33}, \quad (4)$$

$$\varepsilon'_{xy} = \varepsilon_r(\lambda_{11}\lambda_{21} + \lambda_{12}\lambda_{22})/\lambda_{33} = \varepsilon'_{yx}, \quad (5)$$

$$\varepsilon'_{zz} = \varepsilon_r/k_0, \quad \varepsilon'_{xz} = \varepsilon'_{yz} = \varepsilon'_{zx} = \varepsilon'_{zy} = 0, \quad (6)$$

where  $\lambda_{11} = k_0 - bcy/(ax + by)^2$ ,  $\lambda_{12} = bcx/(ax + by)^2$ ,  $\lambda_{21} = acy/(ax + by)^2$ ,  $\lambda_{22} = k_0 - acx/(ax + by)^2$ , and  $\lambda_{33} = k_0(k_0 - c/(ax + by))^2$ . Obviously, the component  $\varepsilon'_{zz}$  is constant. The magnetic permeability can be expressed in a similar way, and we omit the expressions here. We remark that a 3D transformation have been constructed in equation (2) instead of a 2D transformation to eliminate the singularity of the EM parameters [11,17].

We have assumed that the illusion objects are isotropic:  $\bar{\varepsilon} = \varepsilon_r \bar{I}$  and  $\bar{\mu} = \mu_r \bar{I}$ . For two trapezia regions in Fig. 2(b),  $\varepsilon_r = 3$  and  $\mu_r = 2$ , which generate the illusion of two square dielectric cylinders; for other regions in the illusion medium layer,  $\varepsilon_r = 1$  and  $\mu_r = 1$ , which generate the illusion of free space. Due to the non-conformality of the transformation, some non-diagonal components of the constitutive tensors are non-zero. However, the real fabrication requires the material parameters  $\bar{\varepsilon}'$  and  $\bar{\mu}'$  to be denoted in diagonal tensors. The symmetry of the tensors  $\bar{\varepsilon}'$  and  $\bar{\mu}'$  indicates that there always exists a rotation transformation which maps a symmetric tensor into a diagonal one. Therefore, the material parameters with the eigenbasis can be expressed as

$$\varepsilon_1 = \frac{\varepsilon'_{xx} + \varepsilon'_{yy} - \sqrt{\varepsilon'^2_{xx} - 2\varepsilon'_{xx}\varepsilon'_{yy} + \varepsilon'^2_{yy} + 4\varepsilon'^2_{xy}}}{2}, \quad (7)$$

$$\varepsilon_2 = \frac{\varepsilon'_{xx} + \varepsilon'_{yy} + \sqrt{\varepsilon'^2_{xx} - 2\varepsilon'_{xx}\varepsilon'_{yy} + \varepsilon'^2_{yy} + 4\varepsilon'^2_{xy}}}{2}, \quad (8)$$

$$\varepsilon_z = \varepsilon'_{zz}, \quad (9)$$

and all non-diagonal components are zero. The magnetic permeability  $\mu_1$ ,  $\mu_2$ , and  $\mu_z$  have similar expressions, and are omitted here. We remark that the relation between the virtual and physical coordinates can be established as  $x/x' = y/y' = z/z' = (1 + c/(ax' + by'))/k_0$ .

For TE-wave incidence, only  $\mu_1$ ,  $\mu_2$  and  $\epsilon_z$  are of interest and must satisfy the request of Eqs. (3)-(9). The distributions of  $\mu_1$  and  $\mu_2$  are illustrated in Figs. 3(a)-(b), from which we clearly see that all values are finite without any singularity. It is important to note that the transformation in this illusion device is a compressing mapping, instead of the folding of geometry [10,13,16]. Hence any parameters of the illusion medium layer is not negative. This kind of metamaterials have been extensively studied and fabricated in the experiment of free-space cloaks at microwave frequency [3]. Similar to the experiment of the free-space cloak, such an illusion medium layer is restricted to a narrow frequency band because some parameter components go to zero near the inner boundary [3,15]. The illusion medium layer renders the enclosed metallic square cylinder invisible and projects the illusion of two dielectric square cylinders as shown in Fig. 2(b).

In real applications, artificial metamaterial structures are always lossy. Hence it is interesting to investigate the lossy effect of the medium layer on the illusion property. When we add a loss tangent of 0.01 in both permittivity and permeability of the illusion medium layer, the scattered electric-field distributions (not shown) are very similar to that of two dielectric square cylinders. Hence, the lossy illusion medium layer is still effective.

The presented illusion medium layer could be realized by designing proper metamaterial structures. To achieve the permeability component  $\mu_1$ , we can make use of the split-ring resonators (SRR) [3,11]. One kind of SRR unit, the C structure, has been used for the design, as shown in Fig. 4(a). From the retrieved permeability illustrated in the same figure, we could obtain the permeability component ranging from 0 to 0.7 by adjusting the SRR's size, which satisfies the requirement to  $\mu_1$  shown in Fig. 3(a). Another kind of SRR structure, which can be used to achieve the other permeability component  $\mu_2$ , is demonstrated in Fig. 4(b). Clearly, the retrieved permeability varies from 2 to 5.2 by changing the SRR's size, satisfying the requirement to  $\mu_2$  shown in Fig. 3(b). The component  $\epsilon_z = 2$  or 6 can be realized using the non-resonant structure - I shape [5]. All the imaginary parts of electromagnetic parameters for the metamaterial units at the non-resonant frequencies are small enough to be neglected in the design of illusion medium layer. We remark that it is indeed a complicated work to optimize the overall design to make a compact layout of the illusion medium layer, which will be considered in the further work.

In summary, we have presented a class of optical transformation media, illusion media, which can create one or more virtual objects by using metamaterials. To eliminate the singularity of medium parameters for the illusion device, we construct the transformation in 3D. Hence, in such illusion media, all components of constitutive parameters in the principle coordinate system are finite and positive and they could be realizable using artificial structures.

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## List of Figure Captions

**Fig. 1:** (color online) A simple scheme of an illusion medium layer that transforms the image of an object (a golden apple) into that of the illusion (two green apples). (a) The golden apple (the actual object) enclosed with the illusion medium layer in the physical space. (b) Two green apples (the illusion) in the virtual space.

**Fig. 2:** (color online) The scattered electric-field distributions in the computational domain for (a) a bare metallic square cylinder; (b) a metallic square cylinder with the illusion medium layer; and (c) two dielectric square cylinders when the plane waves are incident horizontally.

**Fig. 3:** (color online) The parameter distributions of the illusion medium layer, (a)  $\mu_1$ , (b)  $\mu_2$ .

**Fig. 4:** (color online) The effective EM parameters versus to the frequency for the metamaterial structures. (a) C structure for  $\mu_1$ ; (b) SRR structure for  $\mu_2$ .

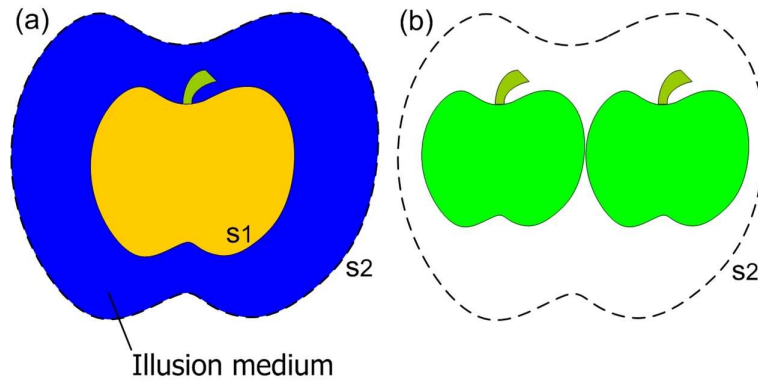


Figure 1:

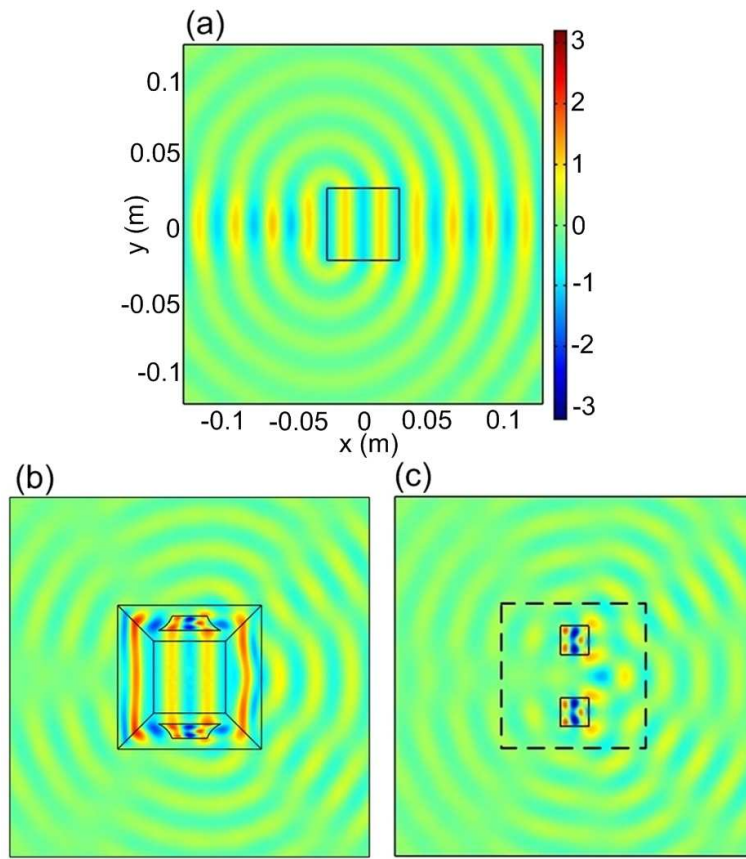


Figure 2:

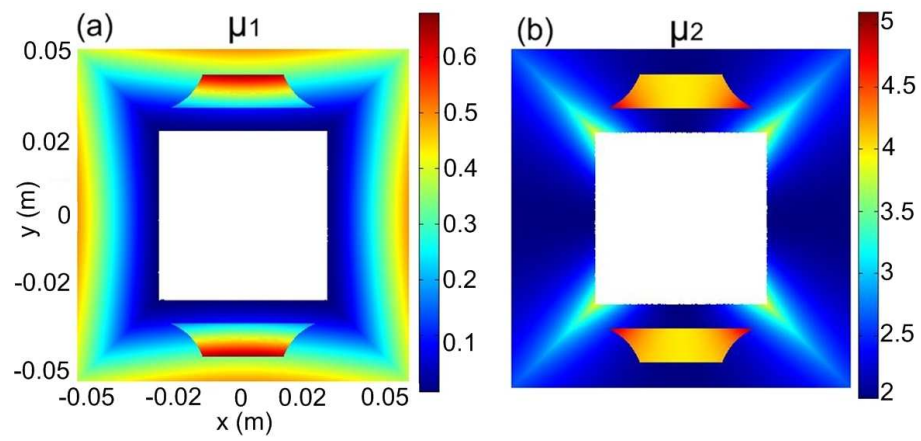


Figure 3:



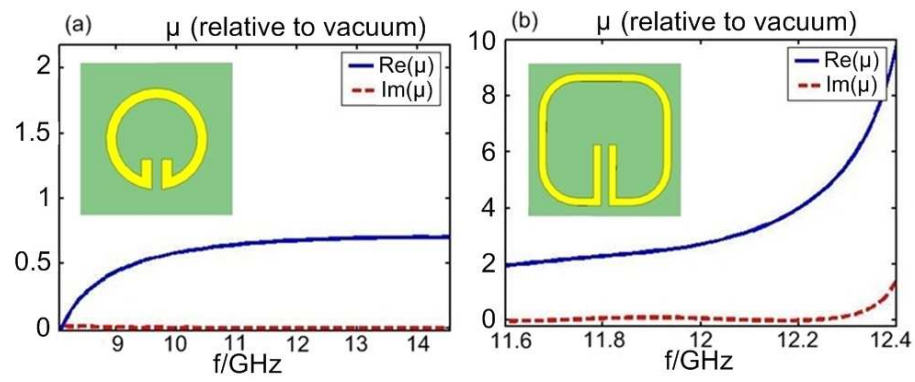


Figure 4: